

Earth and Planetary Materials

Spring 2013

Lecture 10
2013.02.11

Minerals

Long range, 3D internal order

Ions or groups of atoms repeating at regular intervals throughout the mineral

Physical properties

- External form
- Mechanical properties (cleavage)
- Thermal expansion
- Optical properties
- Magnetic + electrical properties

Symmetry

Repetition of objects through rotation, reflection, inversion, and translation

Crystallography

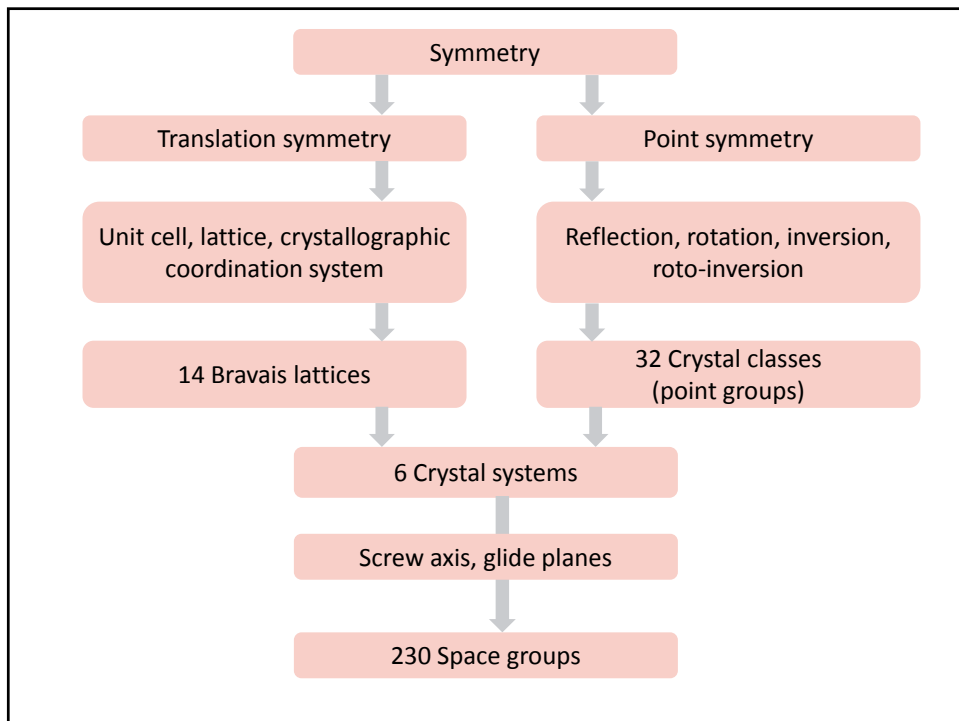
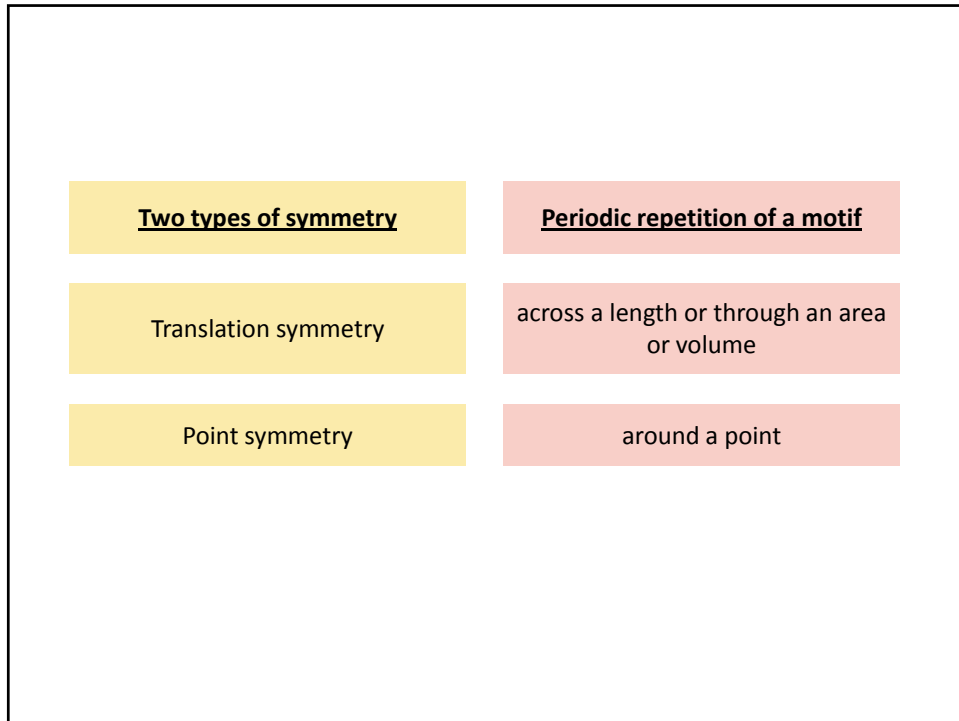
The study of the external form and internal atomic arrangement of crystalline solids, and the principles that govern their growth, external shape, and internal structures.

- Crystalline solids
- Internal atomic arrangement
- Growth
- External form

Symmetry

Describes the periodic repetition of structural features

- Simplifies the description of minerals – we only need to specify a pattern and how it is repeated
- Each mineral has a specific symmetry
 - Minerals with the same chemical composition and different symmetry?



Translational symmetry

Translational symmetry

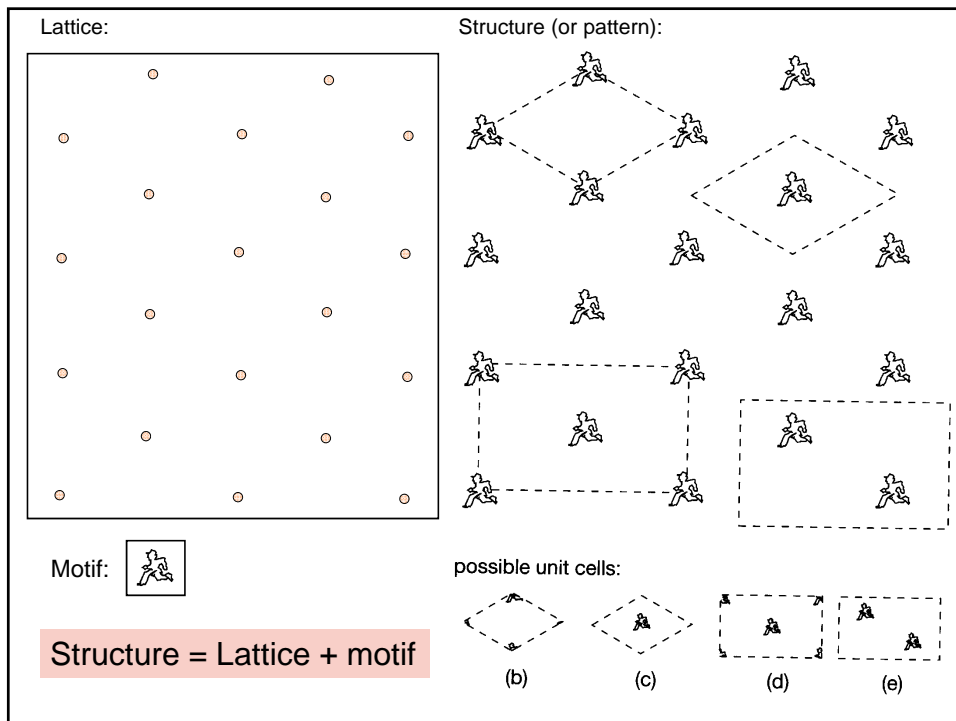
Repetition by movement in equal increments

Unit cell

- The smallest unit of a structure (or pattern) that can be infinitely repeated to generate the entire structure (or to generate the complete print of a pattern)

Structure (or pattern) = motif + lattice

- Motif: Unit of pattern – what is repeated
- Lattice: An imaginary pattern of points in which every point has an environment that is identical to that of every other point –
How to repeat
 - No specific origin as it can be shifted parallel to itself



Size of a unit cell

- Atoms along the outside of the unit cell are shared with adjacent cells
 - interior atoms count as 1 each
 - atoms on corners count as 1/8 (each corner shared among eight cells)
 - atoms on edges count as 1/4 (shared among four cells)
 - atoms on faces count as 1/2 (shared between two cells)

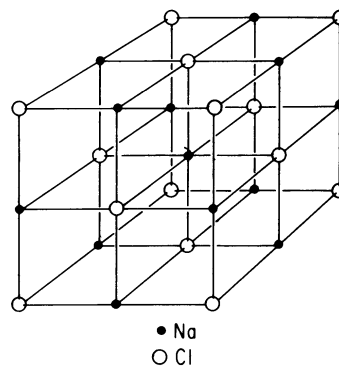
Example:

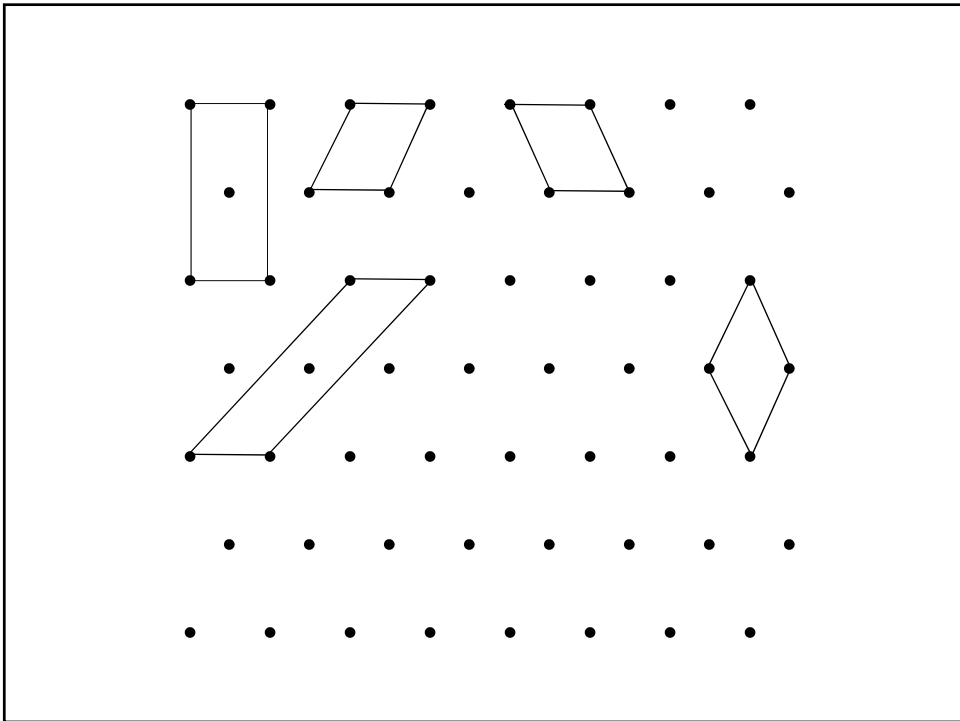
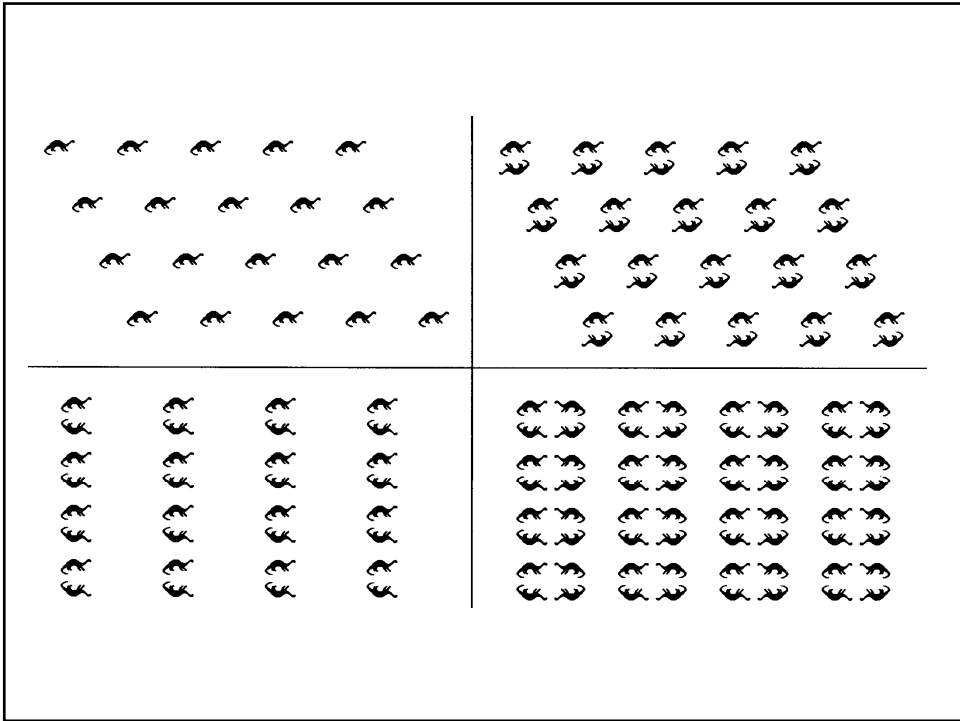
Count Na (filled symbols):

1 interior (in center)

12 edges * 1/4 = 3

Total = 1 + 3 = 4 Na



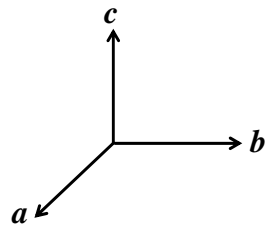


Crystallographic coordinate system

- Unique to a mineral and forms the edges of its unit cell
- Crystallographic axes
- Crystallographic angles

Crystallographic axes

- All other systems: a, b, c
Hexagonal system: a_1, a_2, a_3, c
- Each axis is of different length
- If they are equal, then label as a_1, a_2 , etc
- Ends of axes are designated + or –

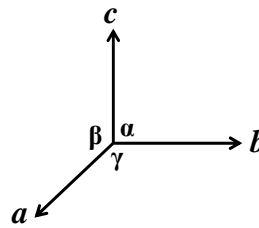


Crystallographic angles

α = between ***b*** and ***c***

β = between ***a*** and ***c***

γ = between ***a*** and ***b***



Bravais lattices

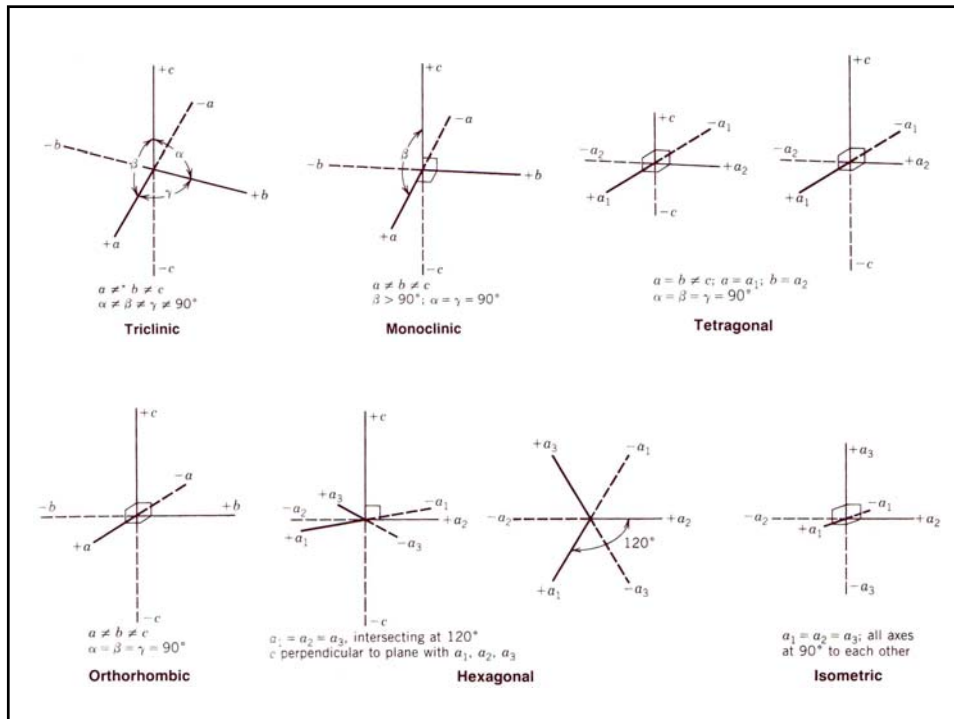
The number of possible lattices is limited

French crystallographer Auguste Bravais (1811-1863):

In three-dimensional space only 14 different lattices may be constructed – Bravais lattices

The 14 Bravais lattices can be divided among 6 crystal systems, each corresponding to a type of coordinate system

| Crystal systems | Coordination system |
|--------------------|--|
| Triclinic | $a \neq b \neq c, \alpha \neq \beta \neq \gamma \neq 90^\circ$ |
| Monoclinic | $a \neq b \neq c, \alpha = \gamma = 90^\circ \neq \beta$ |
| Orthorhombic | $a \neq b \neq c, \alpha = \beta = \gamma = 90^\circ$ |
| Tetragonal | $a = b \neq c, \alpha = \beta = \gamma = 90^\circ$ |
| Trigonal/Hexagonal | $a = b \neq c, \alpha = 90^\circ, \gamma = 120^\circ$ |
| Cubic | $a = b = c, \alpha = \beta = \gamma = 90^\circ$ |



The 14 Bravais lattices are furthermore of 3 different types

- Primitive lattice (P)
 - Lattice point at each corner of the three-dimensional unit cell

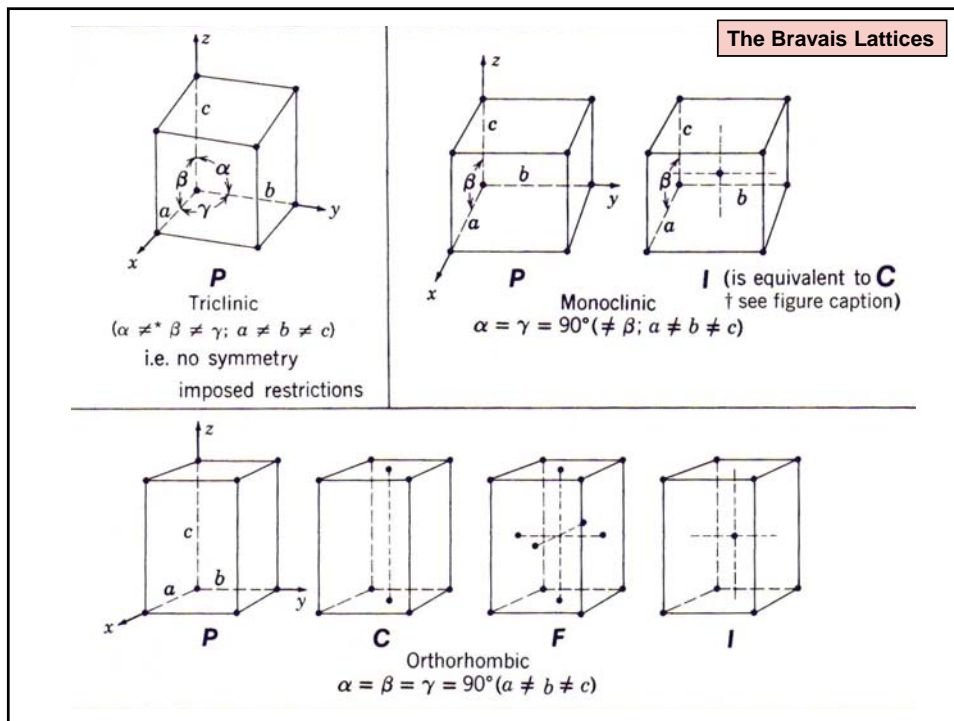
- Body-centered lattice (I)
 - Lattice points at each corner of the unit cell
 - A lattice point at the center of the three-dimensional unit cell

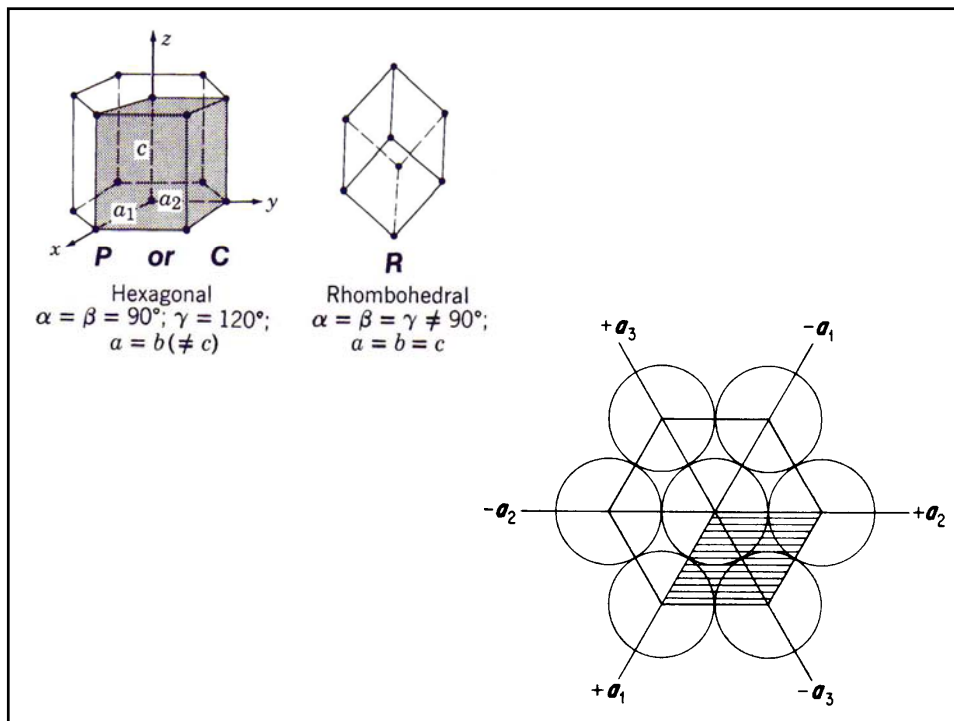
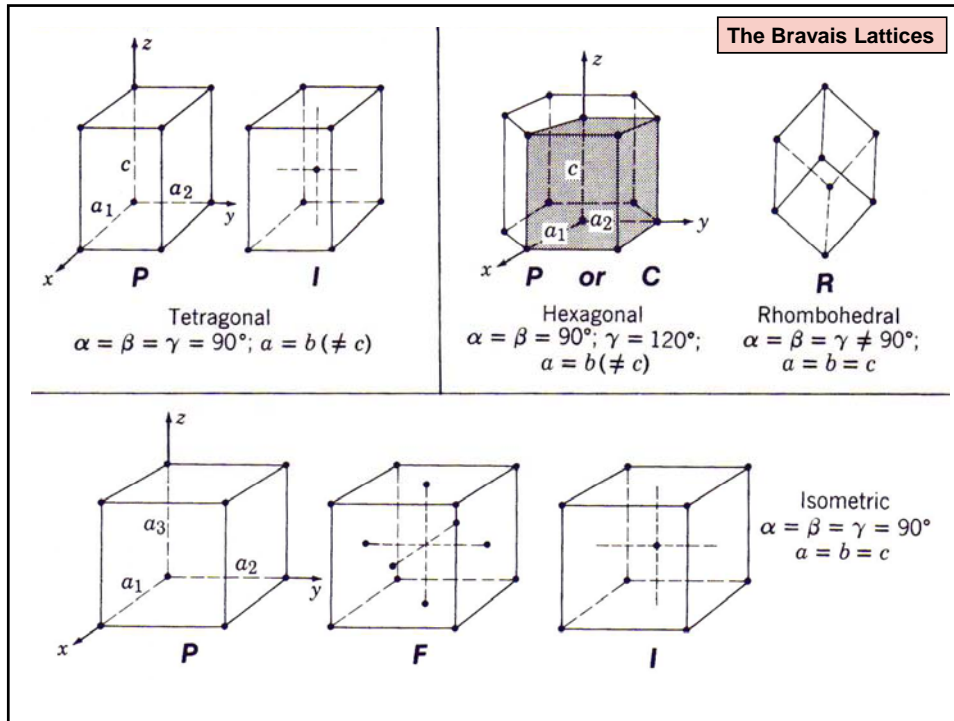
- Face-centered lattice (F or C)
 - Lattice points at each corner of the unit cell
 - Lattices points at either the centers of just one pair of faces (C) or the centers of all three pairs of faces (F)

| Crystal systems | Coordination system | Lattice types |
|--------------------|--|---------------|
| Triclinic | $a \neq b \neq c, \alpha \neq \beta \neq \gamma \neq 90^\circ$ | P |
| Monoclinic | $a \neq b \neq c, \alpha = \gamma = 90^\circ \neq \beta$ | P, C |
| Orthorhombic | $a \neq b \neq c, \alpha = \beta = \gamma = 90^\circ$ | P, C, I, F |
| Tetragonal | $a = b \neq c, \alpha = \beta = \gamma = 90^\circ$ | P, I |
| Trigonal/Hexagonal | $a = b \neq c, \alpha = 90^\circ, \gamma = 120^\circ$ | P, R |
| Cubic | $a = b = c, \alpha = \beta = \gamma = 90^\circ$ | P, I, F |

Cell centering symbols

- P: primitive
- F: face centered (extra lattice point on each face)
- I: body centered (lattice point in the center of cell)
- C: c-centered (lattice point on the plane normal to c);
B or A centering also possible
- R: rhombohedral





Planes and Miller indices (hkl)

- We need a way to identify planes in crystals
- Some planar objects of interest:
 - crystal faces
 - cleavage planes
- Planes are identified by Miller indices (hkl)

Miller indices (hkl)

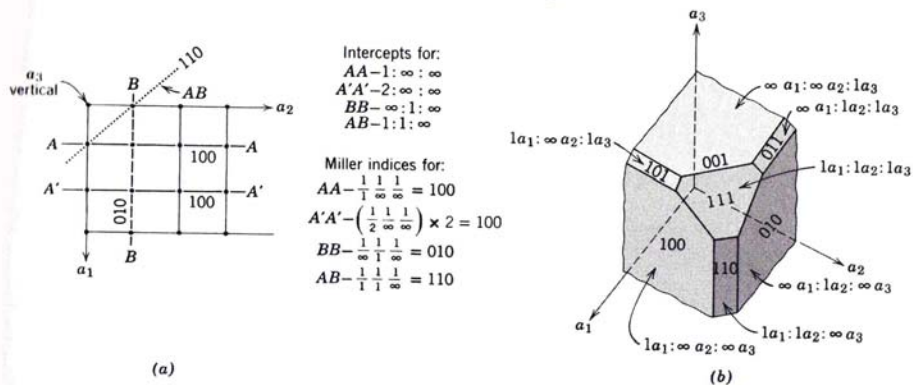
The Miller indices of a face consist of a series of whole numbers that have been derived from the intercepts by inverting and, if necessary, the subsequent clearing of fractions.

- Always given by parentheses

How to get Miller indices

- Write the intercepts of the plane in units of the cell edge lengths
- Invert the intercepts
- Clear fractions
- Remember that $1/(\text{infinity}) = 0$
- Negative indices are given by a "bar" over the number

FIG. 6.29 (a) A plan view showing intercepts and Miller indices for some planes in an isometric lattice. (b) Intercepts and Miller indices of some faces modifying the corner and edges of a cube.



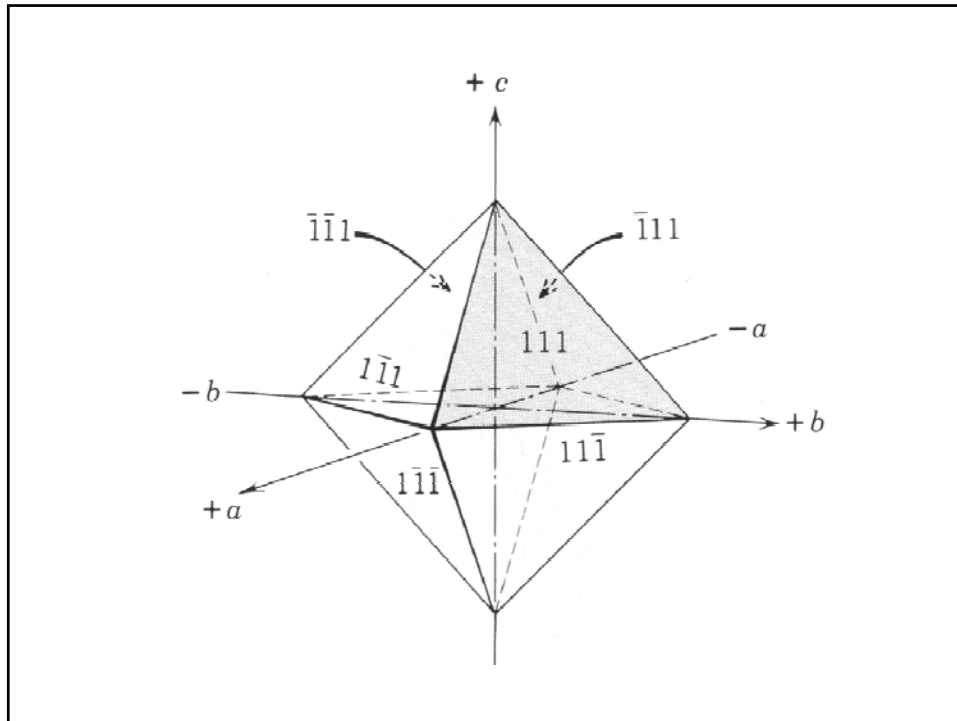
- Example: plane intercepts at 2 units along a ; 3 units along b , but is parallel to c

Intersects: 2, 3, ∞

Inversion: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{\infty}$

Miller index: (3 2 0)

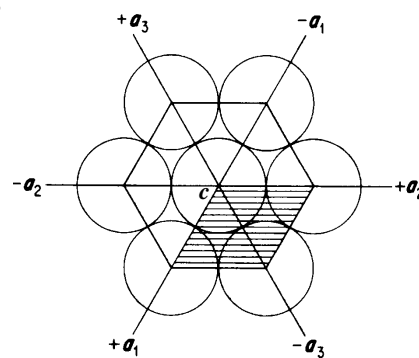
- Example: $(1\bar{1}0)$ is the “one bar one oh” plane

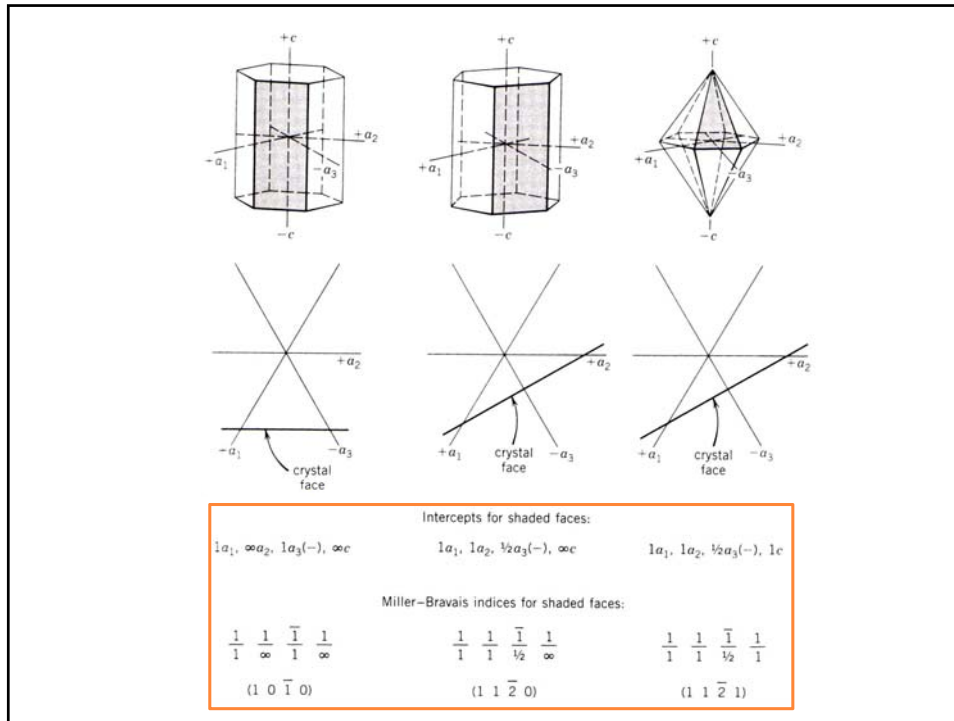


- Hexagonal system: $(h \ k \ \bar{l})$

$$a_1, a_2, a_3, c$$

- Bravais- Miller system of indexing





Lines $[u\ v\ w]$

Zone

a collection of faces with parallel edges

Zone axis

- A line through the center of the crystal
- Parallel to the lines of face intersections
- Always in square brackets: $[u\ v\ w]$
 - u = number of units along a
 - v = number of units along b
 - w = number of units along c

Example:

- $[3\ 2\ 0]$ corresponds to a line from the origin through the point $(3a, 2b, 0c)$
- This line is perpendicular to c

Special lines

- $[1\ 0\ 0]$ is the a axis
- $[0\ 1\ 0]$ is the b axis
- $[0\ 0\ 1]$ is the c axis

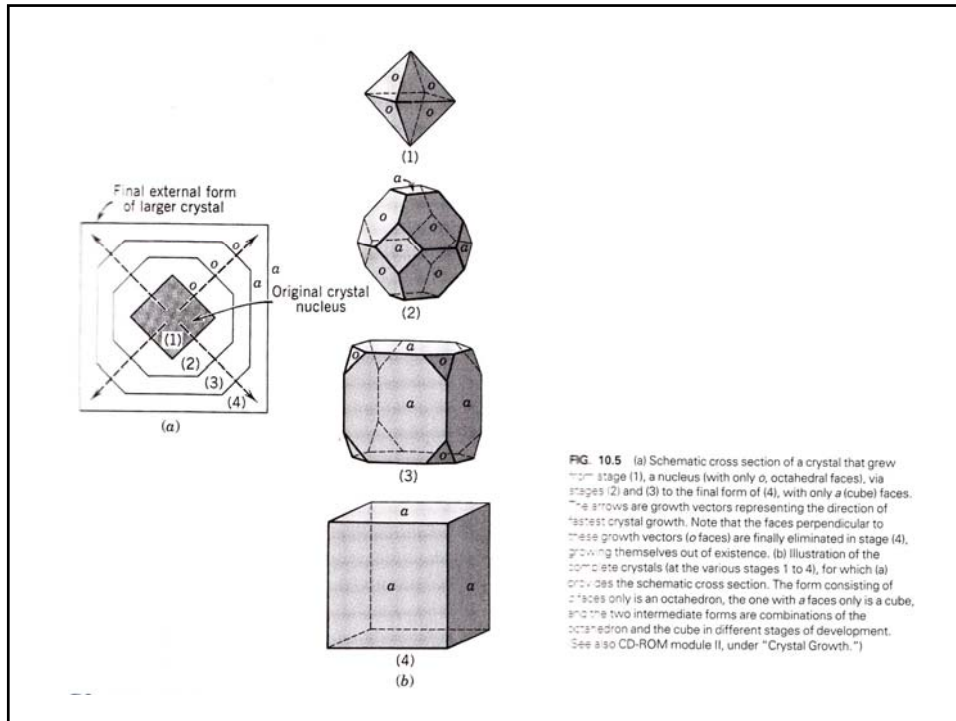


FIG. 10.5 (a) Schematic cross section of a crystal that grew from stage (1), a nucleus (with only *o*, octahedral faces), via stages (2) and (3) to the final form of (4), with only *a* (cube) faces. The arrows are growth vectors representing the direction of fastest crystal growth. Note that the faces perpendicular to these growth vectors (*o* faces) are finally eliminated in stage (4), growing themselves out of existence. (b) Illustration of the complete crystals (at the various stages 1 to 4), for which (a) provides the schematic cross section. The form consisting of *o* faces only is an octahedron, the one with *a* faces only is a cube, and the two intermediate forms are combinations of the octahedron and the cube in different stages of development. See also CD-ROM module II, under "Crystal Growth."

Calcite crystals

Calcite crystals,
Forms: r {0112} and f {0221},
 negative rhombohedrons,
 $+$ {1011} and M {4041}
 positive rhombohedrons,
 m {1010}, prisms,
 e {0001}, basal pinacoid,
 s {2131}, scalenohedron

